

Constant Dilaton Vacua and Kinks in 2D (Super-)Gravity

Luzi Bergamin

*Institute for Theoretical Physics,
 Vienna University of Technology, 1040 Vienna, Austria,
 email: bergamin@tph.tuwien.ac.at*

Abstract

2D dilaton (super-)gravity contains a special class of solutions with constant dilaton, a kink-like solution connecting two of them was recently found in a specific model that corresponds to the KK reduced 3D Chern-Simons term. Here we develop the systematics of such solutions in generalized 2D dilaton gravity and supergravity. The existence and characteristics thereof essentially reduce to the discussion of the conformally invariant potential W , restrictions in supergravity come from the relation $W = -w^2$. It is shown that all stable kink solutions allow a supersymmetric extension and are BPS therein. Some examples of polynomial potentials are presented.

Keywords: 2D dilaton gravity, kink solutions, supergravity.

1 Introduction

Recently the study of the Kaluza-Klein reduced 3D gravitational Chern-Simons term attracted attention due to an interesting kink solution originally found in ref. [13] (cf. e.g. refs. [1,11,14], an unexpected relation to 4D gauged $N = 2$ supergravity has been found in ref. [8]). It was shown in ref. [13] that within a certain conformal frame the KK reduced Chern-Simons term leads to the action

$$\mathcal{S} = -\frac{1}{8\pi^2} \int d^2x \sqrt{-g}(FR + F^3), \quad (1)$$

where R is the curvature scalar and F the dual field strength tensor of the $U(1)$ gauge field. In that paper three types of solutions to the action (1) were presented, the first one with $R = c$, $F = 0$, the second one with $R = -2c$, $F = \pm\sqrt{c}$ and a third one given the interpretation of a kink connecting two solutions of the second type with different signs of F . In ref. [11] it was pointed out that eq. (1) can be reformulated as a first order action of generalized dilaton gravity (cf. [12] and refs. therein), which immediately allows to discuss *all* classical solutions at a global level. This also allowed a straightforward supersymmetrization [1] and it was shown that the kink solution is a BPS state therein.

The reformulation in ref. [11] suggests that kink-like solutions as found in ref. [13] exist in many other models of generalized 2D dilaton gravity as well. It is the purpose of this paper to examine the questions of existence and behavior thereof. In section 2 we define the kink as a certain solution of generalized dilaton gravity and discuss its basic properties in the bosonic model as well as within the supersymmetric extension. The stability of these solutions is addressed in section 3 and in section 4 polynomial potentials are discussed more in detail. Our conclusions are presented in section 5.

2 Constant Dilaton Vacua and Kinks

We consider generalized 2D dilaton gravity in its first order formulation with the action¹ (cf. ref. [12])

$$\mathcal{S} = \int_{\mathcal{M}} \phi d\omega + X^a D e_a + \epsilon(V(\phi) + X^{++}X^{--}Z(\phi)). \quad (2)$$

¹The dilaton ϕ and X^a are scalar fields, ω and e_a are the *independent* spin connection and the zweibein, resp., the latter yields the volume form $\epsilon = \epsilon^{ab}e_b \wedge e_a$. We work in light-cone coordinates labeled as “++” and “--”. The notations and conventions used here are equivalent to the ones used in refs. [4,9].

As one of the unique advantages of this formulation its equations of motion

$$d\phi + X^{++}e_{++} - X^{--}e_{--} = 0, \quad (d\mp\omega)X^{\pm\pm} \pm (V + X^{++}X^{--}Z)e_{\mp\mp} = 0, \quad (3)$$

$$d\omega + \epsilon(V' + X^{++}X^{--}Z') = 0, \quad (d\mp\omega)e_{\pm\pm} + \epsilon X^{\mp\mp}Z = 0 \quad (4)$$

immediately unravel the integrability of the model. Generic solutions [12] are obtained on a patch with $X^{++} \neq 0$ (or equiv. $X^{--} \neq 0$), they are parametrized by the dilaton, X^{++} (X^{--}) and the Casimir function (constant of motion)

$$C = e^Q X^{++} X^{--} + W, \quad Q = \int_{\phi_0}^{\phi} d\varphi Z(\varphi), \quad W = \int_{\phi_0}^{\phi} d\varphi e^Q V(\varphi). \quad (5)$$

In Eddington-Finkelstein gauge the line element is obtained as

$$(ds)^2 = 2 du d\phi + K(\phi, C)(du)^2, \quad K = 2e^Q(C - W) = 2e^Q X^{++} X^{--}, \quad (6)$$

where K is the Killing norm associated to the Killing vector $\partial/\partial u$. This last equation also makes the double nature of ϕ explicit: on the one hand it is a field, the dilaton, on the other hand it may be interpreted as one of the world-sheet coordinates.

There exist solutions of eqs. (3)-(4) with $X^{++} = X^{--} = 0$. If they do not occur at isolated points (bifurcation points) the dilaton must be constant and thus they are called “constant dilaton vacua” (CDV). As can be seen from eq. (3), such CDVs do not exist generically but at the roots of the potential V only, $V(\phi_{CDV}) = 0$. The first equation in (4) tells us that the curvature is constant as well, as

$$d\omega = -\epsilon V'(\phi_{CDV}), \quad R = -2 * d\omega = 2V'(\phi_{CDV}). \quad (7)$$

The geometric structure of CDVs is thus AdS space ($R < 0$), dS space ($R > 0$) or Minkowski or Rindler space ($R = 0$). Finally the Casimir function is simply the value of the conformally invariant potential $W(\phi_{CDV})$. Notice that the value of Z (the conformal frame) is irrelevant and thus CDVs do not change under globally regular conformal transformations.

2.1 Kinks in Bosonic Models

The discussion of CDVs is important in the current context, as the three solutions with constant curvature of the action (1) found in ref. [13] exactly correspond to the three CDVs of the potential [11]

$$V \propto \phi^3 - c\phi \quad (c > 0), \quad \phi_1 = -\sqrt{c}, \quad \phi_2 = 0, \quad \phi_3 = \sqrt{c}. \quad (8)$$

The kink solution belongs to the class of generic solutions discussed above and approaches the CDV solutions at the points $\phi = \phi_1$ and $\phi = \phi_3$.

We generalize this concept to arbitrary potentials $V(\phi)$ with CDVs $\phi_1 < \phi_2 \dots < \phi_n$ and define the kink as a solution of (3)-(4) that approaches the CDV solutions at the points ϕ_1 (the initial point ϕ_I) and ϕ_n (the final point ϕ_F), i.e. X_{kink}^{++} and X_{kink}^{--} vanish at these points. The remaining CDV points $\phi_2 \dots \phi_{n-1}$ lie in the “interior” of the kink and the X_{kink}^a do not necessarily vanish there.

To discuss the systematics of such kinks we have to address the question how different CDVs occur and how a kink solution connecting two of them looks like. The key observation is that this basically boils down to a discussion of the function $W(\phi)$. As long as the conformal factor $\exp(Q)$ remains regular and non-zero (what we will assume in the following) CDVs occur at local extrema or turning points of W with

$$W'' > 0 \Leftrightarrow \text{dS space}, \quad W'' < 0 \Leftrightarrow \text{AdS space}, \quad W'' = 0 \Leftrightarrow \text{Minkowski or Rindler space}.$$

The value of the Casimir function is of interest as well. As it is a constant of motion, kink solutions only can exist between two CDVs with the same value of thereof: $W(\phi_I) = W(\phi_F) \equiv C$. Moreover, also the Killing norm is determined by the conformally invariant potential according to eq. (6), in a kink solution horizons occur wherever $W(\phi_h) = W(\phi_I)$. Thus it is seen that—up to the exact value of the curvature scalar—all information about CDVs and possible kinks is stored in the function $W(\phi)$.

From these observations we can read off the following immediate consequences:

- An AdS and a dS CDV are never neighboring.
- The existence of kink solutions requires at least three CDVs, an AdS-dS kink at least four CDVs. The simplest kink is AdS-dS-AdS (or dS-AdS-dS) and realized e.g. in the model of ref. [13].
- At the initial and final points a kink always approaches an extremal horizon.
- An AdS-AdS (dS-dS) kink always has an even number, an AdS-dS kink always an odd number of horizons. Here extremal horizons must be counted according to their multiplicity, i.e. extrema of W count with an even number, turning points with an odd number. In particular it follows that an AdS-dS kink always has at least one horizon in its interior.
- The number of horizons in the interior of a kink is limited by the number of CDVs, in particular

$$\#(\text{CDVs}) \geq \#(\text{NEH}) + 2\#(\text{EH}) + 1 , \quad (9)$$

where NEH stands for “non-extremal horizons” and EH for “extremal horizons”, resp., and the CDVs/horizons at ϕ_I and ϕ_F are not counted. Extremal horizons count twice as they can occur at the points $\phi_2 \dots \phi_{n-1}$ only, i.e. an extremal horizon always requires the existence of a CDV with $W(\phi_{CDV}) = C$.

2.2 Kinks in Supergravity

Supersymmetric extensions of generalized dilaton gravity have been known for some time [16], its first order formulation, i.e. the supersymmetric extension of the action (2), has been formulated in refs. [4, 5, 9]. As in the bosonic model all classical solutions are easily derived therefrom.

Supersymmetry imposes a condition on the form of the potential $V(\phi)$ as it must be expressible in terms of a prepotential $u(\phi)$, which appears in the spinorial part of the action²:

$$V = -\frac{1}{8}((u^2)' + u^2 Z) \quad W = -2w^2 \quad w = \frac{1}{4}e^{Q/2}u \quad (10)$$

There exist two different types of CDVs, namely³

$$u(\phi_{CDV}) = 0 = w(\phi_{CDV}) \quad \rightarrow \quad C = 0 \quad R = -8e^{-Q}(w')^2 \leq 0 , \quad (11)$$

$$\left(\frac{u'}{u} + \frac{Z}{2}\right)\bigg|_{\phi=\phi_{CDV}} = 0 = w'(\phi_{CDV}) \quad \rightarrow \quad C = -2w^2 \leq 0 \quad R = -8e^{-Q}ww'' . \quad (12)$$

Except for the special case $w = w' = 0$, a kink always connects two $w = 0$ or two $w' = 0$ CDVs, mixed kinks do not exist. Further between two $w = 0$ CDVs there must be at least one with $w' = 0$.

As W is bounded from above ϕ_I and ϕ_F can never be dS CDVs, which are local minima of W . Therefore in supergravity dS-dS and AdS-dS kinks do not exist. This is obvious for $w = 0$ kinks but according to our definition in section 2.1 it holds for $w' = 0$ kinks as well. Further characteristics of the two types of kinks are:

$w = 0$ kinks: All (interior) horizons are maxima of W and thus extremal. All $w = 0$ CDVs are at the same time horizons and vice versa. $w = 0$ kinks are BPS states (cf. refs. [1, 2]).

$w' = 0$ kinks ($w \neq 0$): They can have extremal and non-extremal horizons in its interior. The extremal ones are $w' = 0$ CDVs. These kinks are never BPS.

It is interesting to consider the relation (9) in supergravity. For both types of kinks one obtains (ϕ_I and ϕ_F are not counted)

$$\#(w = 0 \text{ CDVs}) \leq \#(w' = 0 \text{ CDVs}) - 1 . \quad (13)$$

For BPS kinks this is a simple rephrasing of the relation (9) for the special case where all horizons are extremal.

²The complete supergravity action is not presented here, as we consider solutions with vanishing fermionic fields, only. The complete action and its solutions have been presented in [4].

³The Casimir function is defined up to an additive constant, only. In contrast to the bosonic model there exists a preferred choice in supergravity by demanding that BPS states have $C = 0$ [2].

More interesting is the non-BPS kink. As ϕ_I and ϕ_F cannot be dS, ϕ_2 and ϕ_{n-1} are not $w = 0$ CDVs which yields the “−1” in (13). If ϕ_I or ϕ_F are $R = 0$ CDVs it is simpler to replace (13) by

$$\#(w = 0 \text{ CDVs}) \leq \#(w' = 0 \text{ CDVs}) - 3, \quad (14)$$

where *all* CDVs are counted according to their multiplicity. On top of this (9) is still a non-trivial inequality for this type of kinks.

The first order formulation allows the discussion of extended supergravity as well, in particular the $N = (2, 2)$ version as shown in refs. [6, 7]. This appears to yield an especially interesting extension, as the field content contains an additional scalar field, the scalar partner of the $U(1)$ gauge field. More specifically, one can define a complex “dilaton” $X = \phi + i\pi$ and supersymmetry restricts $w(X)$ to be an analytic function in X [7], which could result in unique advantages in the discussion of polynomial potentials. However, it turns out that all $N = (2, 2)$ kinks can be realized in the bosonic model already, as the existence of CDV solutions does not yet imply the existence of a kink. Indeed, the complex “dilaton” remains a cross: its real part can be interpreted as a coordinate while the imaginary part is simply a constant of motion⁴ (Casimir function), namely the charge. Thus a kink can only exist between two CDVs with the same charge, which in addition always can be redefined in such a way that it vanishes on the kink solution⁵.

3 Stability of the Kink

If the kink shall describe the non-trivial configuration of a certain vacuum state, we should worry besides its mere existence about its stability as well. Two different criteria are discussed here: thermodynamical stability and energy considerations. We start with the thermodynamical stability, which will turn out to yield a less restrictive bound.

All our kink solutions exhibit at least two (Killing) horizons and thus we may ask about the Hawking effect. The Hawking temperature can be calculated as a purely geometric quantity from surface gravity (cf. e.g. [17]) as

$$T_H = \frac{1}{4\pi} \left| \frac{d}{d\phi} K(\phi, C) \right|_{\phi=\phi_h} = \frac{1}{2\pi} |V(\phi)|_{\phi=\phi_h}. \quad (15)$$

Here, ϕ_h is the value of the dilaton at the horizon, the prefactor is subject to conventions, but remains unimportant in the following discussion. Not surprisingly the Hawking temperature vanishes if ϕ_h is at the same time a CDV of V , i.e. if the horizon is extremal. Therefore, the outer horizons at ϕ_I and ϕ_F always have vanishing Hawking temperature. However, this need not be true for eventual interior horizons. But in that case we encounter a thermodynamical instability as there are always at least two neighboring horizons with different temperatures. Of course, these considerations are rather academic in the current context. Our model does not have any physical degrees of freedom and thus there cannot be any thermal radiation. A more elaborate study of thermodynamical stability should thus start from the kink coupled to matter fields, in that context also a more reliable determination of the Hawking temperature is possible (cf. ref. [12]).

As noted above, kink-like solutions are usually of interest if they represent a non-trivial vacuum configuration. Thus we should look for kinks that are solutions of lowest energy. A simple definition of energy is possible in 2D dilaton gravity thanks to the Casimir function defined in eq. (5): It is essentially equivalent to (minus) the ADM mass wherever the latter is defined and it yields a conserved quantity in any space-time and—with an appropriate completion—even after the coupling of dynamical degrees of freedom [10, 15] (for supersymmetric theories cf. ref. [2]). In our case we thus may define the energy as

$$M = -C = -W|_{\phi=\phi_h}. \quad (16)$$

Therefore the kink is stable if W is bounded from above and if $W(\phi_h)$ represents the maximum of W . Then no horizon can occur for $M < M_{\text{kink}}$ and we would encounter naked singularities. Excluding the latter we find that the kink is a solution of lowest energy. Of course, the condition is automatically satisfied for all horizons of the kink if it is satisfied for one of them.

⁴This is correct for bosonic field configurations, only. Non-trivial fermionic fields yield soul contributions to the charge [7], which leads to a subtle modification of the solutions [6].

⁵In contrast to the mass C there exists no preferred choice to fix the arbitrary constant in the definition of the charge, as BPS states exist for any value thereof [6].

Obviously the above requirement is easily translated into the condition

$$W(\phi) = -2w(\phi)^2, \quad (17)$$

where the numerical factor has been chosen in such a way that it coincides with (10). Not surprisingly we find that all BPS kinks are stable (for the specific example of the kink of ref. [13] this has been observed in ref. [1] already). But the converse is true as well: Any stable kink solution allows a supersymmetric extension and therein it is BPS. Accordingly all horizons of a stable kink are extremal and therefore the thermodynamical stability considered above is automatically guaranteed.

4 Polynomial Potentials

To illustrate the general features of the kinks derived in the previous section we present a few examples. As a first simplification we set the conformal factor to unity ($e^Q = 1$, $Z = 0$), which does not change any important physical behavior of the kink⁶. Furthermore we simplify our considerations by choosing polynomials with a maximal number of real roots. By rescaling the dilaton field, the coefficient of the leading term can be set to ± 1 and furthermore the shift-invariance is used to set $\phi_F = -\phi_I = \phi_A$. We can make an ansatz of this type either for V or for W . Potentials V of odd degree describe dS-dS (“+”-sign) or AdS-AdS (“-”-sign) kinks, even degrees dS-AdS or AdS-dS kinks resp. We restrict our discussion to the case with the negative sign, as this situation alone can yield stable kink solutions and thus be extended to supergravity. All results on the bosonic kinks generalize straightforwardly to the positive sign. The choice

$$V = -(\phi + \phi_A)(\phi - \phi_2) \dots (\phi - \phi_{n-1})(\phi - \phi_A) \quad (18)$$

has the advantage that all CDVs together with the associated curvatures

$$R(\phi_i) = -2 \prod_{j \neq i} (\phi_i - \phi_j) \quad (19)$$

are immediate. However the positions of the horizons in general cannot be determined analytically. This may be seen as a minor problem for the interior horizons, but if the condition $W(\phi_A) = W(-\phi_A)$ cannot be solved we are unable to determine whether the potential (18) actually has a kink solution or not. There exists one class of potentials that does not suffer of this problem, the symmetric AdS-AdS (or dS-dS) kink. It is described by

$$V = -(\phi + \phi_A)(\phi + \phi_2) \dots (\phi + \phi_n)\phi(\phi - \phi_n) \dots (\phi - \phi_A), \quad (20)$$

which automatically satisfies $W(\phi_A) = W(-\phi_A)$. The relation (19) simplifies to

$$R(\phi_i) = R(-\phi_i) = -4\phi_i^2 \prod_{j \neq i} (\phi_i^2 - \phi_j^2), \quad R(0) = 2(-1)^{n+1} \prod_i \phi_i^2. \quad (21)$$

Unfortunately we are not able to determine the number and the positions of interior horizons. As is seen from eq. (9) even a large number of CDVs cannot guarantee the existence of interior horizons.

Instead of (20) the more restrictive ansatz

$$W = -(\phi + \phi_A)(\phi - \phi_2) \dots (\phi - \phi_{n-1})(\phi - \phi_A) \quad (22)$$

can be made. Here ϕ_A still refers to the “asymptotic” CDVs as in (18), but the other ϕ_i now label the positions of the horizons instead of the CDVs. Therefore this ansatz yields $2n$ horizons while the corresponding V generates $2n - 1$ CDVs, which corresponds to the equality in eq. (9). This way the causal structure of the solution becomes obvious however the positions and characteristics (curvatures) of the different CDVs are not available in general.

Also in supersymmetric theories two different ansätze can be made, either for the prepotential w or its derivative. By choosing

$$w = (\phi + \phi_A)(\phi - \phi_2) \dots (\phi - \phi_{n-1})(\phi - \phi_A) \quad (23)$$

⁶It is important to notice that conformal transformations do change the physics, if they are not globally well defined. However, such conformal transformations would destroy the kink [1].

with eq. (10) the supersymmetric version of eq. (22) is obtained. According to the discussion above the ϕ_i label the horizons and at the same time all $w = 0$ CDVs. They are characterized by the curvatures

$$R(\phi_i) = -4 \prod_{j \neq i} (\phi_i - \phi_j)^2 . \quad (24)$$

All kink solutions from eq. (23) are stable and within supergravity they are BPS states. A non-BPS kink is possible only if w does not have the maximal number of real roots, which is always possible according to eq. (13).

4.1 The Simplest Kink

For illustrational purposes potentials of order three, four and five are considered more in detail. A kink solution requires at least three CDVs, therefore the simplest kink is described by the potential

$$V = -(\phi + \phi_A)(\phi - \phi_2)(\phi - \phi_A) . \quad (25)$$

The existence of a kink solution requires $W(\phi_A) = W(-\phi_A)$ and therefore $\phi_2 = 0$. By an appropriate choice of the integration constant in eq. (5) W can be written as

$$W = -\frac{1}{4}\phi^4 + \frac{1}{2}\phi_A^2\phi^2 - \frac{1}{4}\phi_A^4 = -\frac{1}{4}(\phi^2 - \phi_A^2)^2 . \quad (26)$$

This is exactly the kink found in refs. [11, 13], to this order the kink is unique. All models allow a supersymmetric extension and the kink is BPS therein [1]. For any further discussions we refer to refs. [1, 11, 13].

4.2 The Simplest AdS-dS kink

An AdS-dS kink requires at least four CDVs and has at least one horizon in its interior. The simplest potential is thus

$$V = -(\phi + \phi_A)(\phi - \phi_2)(\phi - \phi_3)(\phi - \phi_A) . \quad (27)$$

A kink solution exists if $\phi_2\phi_3 = -\phi_A^2/5$ and therefore $\phi_2 < 0$, $\phi_3 > 0$. A more stringent restriction comes from the fact that ϕ_2 and ϕ_3 shall lie between $-\phi_A$ and ϕ_A , thus

$$\phi_2 = -\frac{1}{5}\frac{\phi_A^2}{\phi_3} , \quad \phi_A \geq \phi_3 \geq \frac{1}{5}\phi_A . \quad (28)$$

Again we choose the integration constant in eq. (5) in such a way that the kink has $C = 0$. Then it is easily seen that $W(\phi)$ must have five zeros, namely two at $\phi = -\phi_A$ and at $\phi = \phi_A$ resp., and one at the interior horizon $\phi = \phi_h$. Accordingly W can be written as

$$W = -\frac{1}{5}(\phi - \phi_A)^2(\phi + \phi_A)^2(\phi - \phi_h) , \quad \phi_h = \frac{1}{4\phi_3}(5\phi_3^2 - \phi_A^2) . \quad (29)$$

There exist three special choices of ϕ_3 :

$\phi_3 = \phi_A/\sqrt{5}$: This is the “symmetric” kink, $-\phi_2 = \phi_3 = \phi_I$, $|R_A|/|R_I| = \sqrt{5}$, the interior horizon is at $\phi_h = 0$.

$\phi_3 = \phi_A$, $\phi_3 = \phi_A/5$: One of the interior CDVs and the interior horizon coincide with ϕ_A or $-\phi_A$ resp., W has a triple zero there. Thus the AdS CDV ($\phi_3 = \phi_A$) or the dS CDV ($\phi_3 = \phi_A/5$) turn into Minkowski/Rindler space ($R = 0$).

4.3 An Extended Kink

The first two examples provided the simplest possibilities for an AdS-AdS (dS-dS) and a AdS-dS kink, resp. Our last example concerns a non-minimal realization of the AdS-AdS kink with the “minimal non-minimal” ansatz

$$V = -(\phi + \phi_A)(\phi - \phi_2)(\phi - \phi_3)(\phi - \phi_4)(\phi - \phi_A) . \quad (30)$$

The condition $W(\phi_A) = W(-\phi_A)$ yields the constraint

$$\frac{1}{5}(\phi_2 + \phi_3 + \phi_4)\phi_A^2 = -\phi_2\phi_3\phi_4. \quad (31)$$

There exist three possibilities for the causal structure of this solution: The kink can have no interior horizon ($W(\phi_3) < W(\phi_A)$), two interior horizons ($W(\phi_3) > W(\phi_A)$) or one extremal interior horizon ($W(\phi_3) = W(\phi_A)$). For solutions with interior horizons W has the maximal number of six real roots and the positions of the horizons ϕ_{h_1} and ϕ_{h_2} are again straightforwardly deduced as

$$\phi_{h_1} + \phi_{h_2} = \frac{6}{5}(\phi_2 + \phi_3 + \phi_4), \quad \phi_{h_1}\phi_{h_2} = \frac{1}{2}\phi_A^2 + \frac{3}{2}(\phi_2\phi_3 + \phi_2\phi_4 + \phi_3\phi_4). \quad (32)$$

Restricting to stable or equivalently supersymmetric solutions we notice that w is a polynomial of order three. Thus it can have either one or three real roots, the former case obviously does not lead to kink solutions. Therefore we encounter one extremal horizon in the interior of the kink, which is obtained from eq. (32) by imposing $\phi_{h_1} = \phi_{h_2}$, which consequently is also the position of ϕ_3 . The conformally invariant potential becomes $W = ((\phi + \phi_A)(\phi - \phi_3)(\phi - \phi_A))^2$, while V is given by (30) with

$$\phi_2\phi_4 = -\frac{\phi_A^2}{3}, \quad \phi_3 = \frac{3}{2}(\phi_2 + \phi_4). \quad (33)$$

Of course, the special choice $\phi_4 = \phi_A$ implies $\phi_3 = \phi_A$ and W has a fourth order root at ϕ_A . Thus this ansatz also provides the simplest example for a stable kink with one $R = 0$ CDV. Finally we note the simplest stable ($R = 0$)-($R = 0$) kink needs a w of order four, which is at the same time the simplest example where the inequality (13) needs not be satisfied. It is seen that interesting examples of non-minimal kinks immediately lead to polynomials of order higher than four, such that a general analytic treatment is no longer possible.

5 Conclusions

We have presented the systematics of constant dilaton vacua and the associated kink solutions, originally motivated from the KK reduced gravitational Chern-Simons term [11, 13], in 2D generalized dilaton gravity and supergravity. In order that such solutions exist various conditions must be satisfied. This concerns the existence of constant dilaton vacua (they occur at the roots of the potential V , only), but even more importantly a kink can exist only if the value of the conformally invariant potential is the same for the initial and final point of the kink. Restrictions that emerge from supergravity have been discussed and it has been found that all stable kink solutions allow a supersymmetric extension and are BPS therein.

At hand of the $N = (2, 2)$ supersymmetric case it has been shown that an extension of the (scalar) field content does not necessarily lead to a richer structure of kinks. Nevertheless, models with complex dilaton could be an especially interesting topic to consider, if space-time is complexified as well⁷. There exist other interesting extensions of this work. Besides deeper investigations of specific examples many questions of thermodynamics remain open. This concerns the faith of unstable kink configurations as well as problems raised in ref. [1] already, that can be discussed in a broader context now.

Finally the question of asymptotics should be mentioned, discussed in detail in ref. [11] for the specific example of the kink in ref. [13]. As is obvious from its definition given here, the initial and final points of the kink cannot represent spacial infinity. To overcome this problem a patching of (A)dS vacua has been proposed in ref. [11], which however induces matter fluxes at the initial and final horizons. Another way out has been presented in ref. [1]. In any way it should be kept in mind that the kink itself does not represent a global solution of 2D dilaton gravity and thus must be completed in an appropriate way, which of course may depend on the specific application.

Acknowledgment. It is a pleasure to acknowledge stimulating discussions with D. Grumiller, including a collaboration at an early stage of this work. This work has been supported by the project P-16030-N08 of the Austrian Science Foundation (FWF). I would like to thank the organizers of GAS@BS

⁷Complexified space-time in 2D dilaton gravity was considered in [3], however the dilaton remained a real field in that approach.

05 for all their work that made this interesting and fruitful workshop possible. I am especially indebted to R. and M. Poncet for a wonderful stay in Termes (Ardèche, FR), where most of this work has been developed.

References

- [1] L. Bergamin, D. Grumiller, A. Iorio, and C. Nunez. Chemistry of Chern-Simons supergravity: reduction to a BPS kink, oxidation to M-theory and thermodynamical aspects. *JHEP*, 021:0411, 2004.
- [2] L. Bergamin, D. Grumiller, and W. Kummer. Supersymmetric black holes in 2-d dilaton supergravity: baldness and extremality. *J. Phys.*, A37:3881–3901, 2004.
- [3] L. Bergamin, D. Grumiller, W. Kummer, and D. V. Vassilevich. Classical and quantum integrability of 2d dilaton gravities in Euclidean space. *Class. Quant. Grav.*, 22:1361–1382, 2005.
- [4] L. Bergamin and W. Kummer. The complete solution of 2d superfield supergravity from graded Poisson-sigma models and the super pointparticle. *Phys. Rev.*, D68:104005, 2003.
- [5] L. Bergamin and W. Kummer. Graded Poisson sigma models and dilaton-deformed 2d supergravity algebra. *JHEP*, 05:074, 2003.
- [6] L. Bergamin and W. Kummer. Two-dimensional $N = (2,2)$ dilaton supergravity from graded Poisson-sigma models. II: Analytic solution and BPS states. *Eur. Phys. J.*, C39:S53–S63, 2005.
- [7] L. Bergamin and W. Kummer. Two-dimensional $N=(2,2)$ dilaton supergravity from graded Poisson-sigma models I: Complete actions and their symmetries. *Eur. Phys. J.*, C39:S41–S52, 2005.
- [8] Sergio L. Cacciatori, Marco M. Caldarelli, Dietmar Klemm, and Diego S. Mansi. More on BPS solutions of $N = 2$, $d = 4$ gauged supergravity. *JHEP*, 07:061, 2004.
- [9] M. Ertl, W. Kummer, and T. Strobl. General two-dimensional supergravity from Poisson superalgebras. *JHEP*, 01:042, 2001.
- [10] D. Grumiller and W. Kummer. Absolute conservation law for black holes. *Phys. Rev.*, D61:064006, 2000.
- [11] D. Grumiller and W. Kummer. The classical solutions of the dimensionally reduced gravitational Chern-Simons theory. *Ann. Phys.*, 308:211–221, 2003.
- [12] D. Grumiller, W. Kummer, and D. V. Vassilevich. Dilaton gravity in two dimensions. *Phys. Rept.*, 369:327, 2002.
- [13] G. Guralnik, A. Iorio, R. Jackiw, and S. Y. Pi. Dimensionally reduced gravitational chern-simons term and its kink. *Ann. Phys.*, 308:222–236, 2003.
- [14] R. Jackiw. Two-dimensional gravity from three and four dimensions. 2003.
- [15] W. Kummer and P. Widerin. Conserved quasilocal quantities and general covariant theories in two-dimensions. *Phys. Rev.*, D52:6965–6975, 1995.
- [16] Young-Chul Park and Andrew Strominger. Supersymmetry and positive energy in classical and quantum two-dimensional dilaton gravity. *Phys. Rev.*, D47:1569–1575, 1993.
- [17] Robert M. Wald. *General Relativity*. The University of Chicago Press, 1984.